Aero

Sanjay Ghodawat University, Kolhapur 2019-20

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Year and Program:2019-20 S.Y.B.Tech | **School of Technology** | **Department: Aeronautical**

Course Code: AET 201 | **Course Title: Differential Calculus and Transform** | **Semester – III**

Day and Date: Monday 18/11/2019 | **End Semester Examination (ESE)** | **Time:** 10.30 am to 1.30pm **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q 1				
a)	Solve $(D-2)^2 y = 8\left(e^{2x} + \sin 2x + x^2\right)$.	07	L_2	CO1
	OR			
a)	Solve $\left(D^2 - 2D + 1\right) y = x^2 e^{3x}$.	07	L_2	CO1
b)	Solve $x^2 p^2 + y^2 q^2 = z$.	08	L_1	CO2
	OR			
b)	Solve $y^2 q^2 = z\left(z - px\right)$.	08	L_1	CO2
Q.2				
a)	Find the Fourier series of $f(x) = \begin{cases} \pi x & ; 0 \le x \le 1 \\ \pi(2-x) & ; 1 \le x \le 2 \end{cases}$.	07	L_3	CO3
	OR			
a)	Find the Fourier series of $f(x) = e^{-ax}$ in the interval $[-\pi, \pi]$.	07	L_3	CO3
b)	Show that the vector field represented by $\overline{f} = \left(z^2 + 2x + 3y\right) i + \left(3x + 2y + z\right) j + \left(y + 2zx\right) k$ is irrotational but not Solenoidal. Also obtain a scalar function ϕ such that $grad\,\phi = \overline{f}$.	08	L_3	CO5

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OR

b) Given that $\phi = 2x^3y^2z^4$ then show that $\nabla.\nabla\phi = \nabla^2\phi$ 08 L_3 CO5

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplace operator.

Q.3 **Solve any Two of the following.**

a) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$. 08 L_3 CO1

b) Solve $\left(x^2 + y^2\right)(p^2 + q^2) = 1$. 08 L_3 CO2

c) Obtain Fourier series of $f(x) = \begin{cases} 0 & ; -\pi \le x \le 0 \\ x & ; 0 \le x \le \pi \end{cases}$ 08 L_4 CO3

and, hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$.

d) If $\phi = x^3 + y^3 + z^3 - 3xyz$ find i) $r.\nabla\phi$ ii) $div\,\bar{f}$ iii) $curl\,\bar{f}$ 08 L_1 CO5

where $\bar{f} = \nabla\phi$

Q.4 **Solve any Three of the following.**

a) Find Laplace transform of $\int_0^t \frac{e^{-u}\sin u}{u}\,du$. 06 L_2 CO4

b) Find Laplace transform of $e^{-2t}\cosh t.\sin t$. 06 L_2 CO4

c) Using Laplace transform show that $\int_0^\infty e^{-\sqrt{2}t}\frac{\sinh t.\sin t}{t}\,dt = \frac{\pi}{8}$. 06 L_2 CO4

d) Find Laplace transform of $f(t) = \begin{cases} \sin wt & ; o < t < \frac{\pi}{w} \\ 0 & ; \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$. 06 L_3 CO4

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Q.5 **Solve any Three of the following.**

a)	If $f(z)$ is analytic and $\lvert f(z)\rvert$ is constant, prove that $f(z)$ is constant.	06	L_3	CO6
b)	Verify that the real and imaginary parts of $f(z)=e^z$ are harmonic functions.	06	L_3	CO6
c)	Using Cauchy-Riemann equations in polar form prove that $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$.	06	L_3	CO6
d)	Show that the function $u=\cos x.\cosh y$ is harmonic function. Find the harmonic conjugate and corresponding analytic functions.	06	L_4	CO6

Q.6 **Solve any Three of the following.**

a)	Solve by using convolution theorem $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$.	06	L_2	CO4
b)	Solve $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$.	06	L_1	CO4
c)	Evaluate $\int_c \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$, where c is $\lvert z\rvert=1$.	06	L_2	CO6
d)	Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$, where c is $\lvert z\rvert=3$.	06	L_2	CO6

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